In The Name Of God

Iterative Methods For Solving Saddle Point Problems

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Contents

1 - Introduction

We consider the system of linear equations

$$\mathbf{A}\mathbf{u} = \mathbf{b}, \tag{1}$$

where $\mathbf{A} \in \mathbb{C}^{n\times n}$ and $\mathbf{b} \in \mathbb{C}^{n\times n}$. Assume that the matrix $\mathbf{A}$ has a splitting of the following form

$$\mathbf{A} = \hat{\mathcal{P}} + \mathcal{S}, \quad \hat{\mathcal{P}} \in \mathbb{C}^{n\times n} \text{ is PSD and } \mathcal{S} \in \mathbb{C}^{n\times n} \text{ is skew-Hermitian.} \tag{2}$$

This means that the matrix $\mathbf{A}$ is of a positive semidefinite and skew-Hermitian splitting. Let the matrix $\Sigma \in \mathbb{C}^{n\times n}$ be HPD [1].

2 - The EPSS Iteration

Consider the generalized saddle point problems of the form

$$\mathcal{A}u \equiv \begin{bmatrix} A & B^* \\ -B & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix} \equiv b, \qquad (3)$$

where $A \in \mathbb{C}^{n\times n}$ and $C \in \mathbb{C}^{m\times m}$ are nonsymmetric positive semidefinite, $B \in \mathbb{C}^{m\times n}$ $(m \leqslant n)$, $f \in \mathbb{C}^n$ and $g \in \mathbb{C}^m$.

Convernce and Semi Convergence

Let $\mathbf{A}$ ...

Convernce and Semi Convergence

Let $\mathbf{A}$ ...

Theorem

Iterative method ... $\rho(\mathbf{T}) < 1$ where

$$\rho(\mathbf{T}) = \max\{|\lambda|,\ \lambda \in \sigma(\mathbf{T})\}, \tag{4}$$

3 - EPSS Iterative Method

EPSS Iterative method

$\mathbf{A}$ be a PD matrix $(\mathbf{x}^*(\mathbf{A}+\mathbf{A}^*)\mathbf{x} \geq 0, \mathbf{x} \in \mathbb{C}^n)$ and ...

Note

In BTSS TSS PSS HSS and SS ...

Eigenvalues of ...

Let ...

Eigenvalues of ...

Let ...

EPSS method

If matrix $\mathbf{A}$ ...

Example

Consider

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

and $\Sigma = I \quad \mathbf{P} = \mathbf{A}$...

All Splittings

1. $\mathbf{P}_1 = \mathbf{D} + \mathbf{L} + \mathbf{U}^*$.
2. $\mathbf{P}_2 = \mathbf{B_D} + \mathbf{B_L} + \mathbf{B_U}^*$.
3. $\mathbf{P}_3 = \mathbf{H_A} = \frac{1}{2}(\mathbf{A} + \mathbf{A}^*)$.
4. $\mathbf{P}_4 = \mathbf{A}$.

1. Let $x = [1, 1, ..., 1]^T$.
2. Use (20) $GMRES(20)$.
3. ...

Table 1: Itertion and CPU timing for *EPSS* EPSS Iterative method.

	$\Sigma = \alpha \mathbf{Q}_j$	$\mathbf{Q_1} = \mathbf{I}$				$\mathbf{Q_2} = \mathbf{D}$				$\mathbf{Q_3} = \mathbf{BD}$			
n	$\mathbf{P}_i$	$\mathbf{P}_1(TSS)$	$\mathbf{P}_2(BTSS)$	$\mathbf{P}_3(HSS)$	$\mathbf{P}_4(SS)$	$\mathbf{P}_1$	$\mathbf{P}_2$	$\mathbf{P}_3$	$\mathbf{P}_4$	$\mathbf{P}_1$	$\mathbf{P}_2$	$\mathbf{P}_3$	$\mathbf{P}_4$
400	CPU	0.22	0.07	0.07	0.05	0.06	0.03	0.04	0.04	0.04	0.04	0.02	0.05
	IT	40	39	37	77	7	9	7	8	6	8	4	5
800	CPU	0.11	0.06	0.08	0.08	0.07	0.04	0.06	0.06	0.08	0.06	0.04	0.14
	IT	58	55	40	99	8	9	7	8	7	8	4	5
1600	CPU	0.22	0.08	0.12	0.11	0.09	0.07	0.10	0.09	0.13	0.08	0.09	0.35
	IT	80	59	57	143	8	9	7	7	7	8	5	5
3200	CPU	1.33	0.40	0.51	0.34	0.33	0.31	0.47	0.37	0.50	0.38	0.49	1.26
	IT	94	70	73	205	9	10	7	7	8	11	5	5
6400	CPU	3.72	0.41	0.93	0.70	1.55	1.37	0.98	0.72	1.21	0.97	2.01	4.29
	IT	129	73	73	280	8	12	7	6	10	13	5	5

Algorithm

1. Solve $As_1 = x_1$.
2. Let $s_2 = Bs_1 + x_2$.
3. Solve $(\alpha I + BB^T)y_2 = s_2$.
4. Let $y_1 = s_1 - B^T y_2$.

Block

Block

Theorem

Block

Theorem

Lemma

Block

Theorem

Lemma

Remark

Block

Theorem

Lemma

Remark

Proposition

Block

Theorem

Lemma

Remark

Proposition

Note

Block

Theorem

Lemma

Remark

Proposition

Note

Algorithm

Block

Theorem

Lemma

Remark

Proposition

Note

Algorithm

Corollary

Definition

Definition

Proof

4 - References

References

[1] Zhong-Zhi Bai. Block alternating splitting implicit iteration methods for saddle-point problems from time-harmonic eddy current models. *Numerical Linear Algebra with Applications*, 19(6):914–936, 2012.


Thanks For Your
Attention