

In The Name Of God





Iterative Methods For Solving Saddle Point Problems

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We consider the system of linear equations

$$\mathbf{A}\mathbf{u} = \mathbf{b}, \quad (1)$$

where $\mathbf{A} \in \mathbb{C}^{n \times n}$ and $\mathbf{b} \in \mathbb{C}^{n \times n}$. Assume that the matrix \mathbf{A} has a splitting of the following form

$$\mathbf{A} = \hat{\mathcal{P}} + \mathcal{S}, \quad \hat{\mathcal{P}} \in \mathbb{C}^{n \times n} \text{ is PSD and } \mathcal{S} \in \mathbb{C}^{n \times n} \text{ is skew-Hermitian.} \quad (2)$$

This means that the matrix \mathbf{A} is of a positive semidefinite and skew-Hermitian splitting. Let the matrix $\Sigma \in \mathbb{C}^{n \times n}$ be HPD [1].



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Consider the generalized saddle point problems of the form

$$\mathcal{A}u \equiv \begin{bmatrix} A & B^* \\ -B & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix} \equiv b, \quad (3)$$

where $A \in \mathbb{C}^{n \times n}$ and $C \in \mathbb{C}^{m \times m}$ are nonsymmetric positive semidefinite, $B \in \mathbb{C}^{m \times n}$ ($m \leq n$), $f \in \mathbb{C}^n$ and $g \in \mathbb{C}^m$.



Converence and Semi Convergence

Let \mathbf{A} ...

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Converence and Semi Convergence

Let \mathbf{A} ...

Theorem

Iterative method ... $\rho(\mathbf{T}) < 1$ where

$$\rho(\mathbf{T}) = \max \{ |\lambda|, \lambda \in \sigma(\mathbf{T}) \}, \quad (4)$$



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\mathbf{A} be a PD matrix ($\mathbf{x}^*(\mathbf{A} + \mathbf{A}^*)\mathbf{x} \geq 0, \mathbf{x} \in \mathbb{C}^n$) and ...

Note

In BTSS TSS PSS HSS and SS ...



Eigenvalues of ...

Let ...

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Eigenvalues of ...

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If matrix \mathbf{A} ...



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Example

Consider

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

and $\Sigma = I$ $\mathbf{P} = \mathbf{A} \dots$



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All Splittings

1. $\mathbf{P}_1 = \mathbf{D} + \mathbf{L} + \mathbf{U}^*$.
2. $\mathbf{P}_2 = \mathbf{B}_D + \mathbf{B}_L + \mathbf{B}_U^*$.
3. $\mathbf{P}_3 = \mathbf{H}_A = \frac{1}{2}(\mathbf{A} + \mathbf{A}^*)$.
4. $\mathbf{P}_4 = \mathbf{A}$.



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1. Let $x = [1, 1, \dots, 1]^T$.
2. Use (20) $GMRES(20)$.
3. ...



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Table 1: Itertion and CPU timing for *EPSS EPSS Iterative method*.

n	$\Sigma = \alpha Q_j$ P_i	$Q_1 = I$				$Q_2 = D$				$Q_3 = BD$			
		$P_1(TSS)$	$P_2(BTSS)$	$P_3(HSS)$	$P_4(SS)$	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4
400	CPU	0.22	0.07	0.07	0.05	0.06	0.03	0.04	0.04	0.04	0.04	0.02	0.05
	IT	40	39	37	77	7	9	7	8	6	8	4	5
800	CPU	0.11	0.06	0.08	0.08	0.07	0.04	0.06	0.06	0.08	0.06	0.04	0.14
	IT	58	55	40	99	8	9	7	8	7	8	4	5
1600	CPU	0.22	0.08	0.12	0.11	0.09	0.07	0.10	0.09	0.13	0.08	0.09	0.35
	IT	80	59	57	143	8	9	7	7	7	8	5	5
3200	CPU	1.33	0.40	0.51	0.34	0.33	0.31	0.47	0.37	0.50	0.38	0.49	1.26
	IT	94	70	73	205	9	10	7	7	8	11	5	5
6400	CPU	3.72	0.41	0.93	0.70	1.55	1.37	0.98	0.72	1.21	0.97	2.01	4.29
	IT	129	73	73	280	8	12	7	6	10	13	5	5



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Algorithm

1. Solve $As_1 = x_1$.
2. Let $s_2 = Bs_1 + x_2$.
3. Solve $(\alpha I + BB^T)y_2 = s_2$.
4. Let $y_1 = s_1 - B^T y_2$.



Block

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Definition



Proof



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References

- [1] Zhong-Zhi Bai. Block alternating splitting implicit iteration methods for saddle-point problems from time-harmonic eddy current models. *Numerical Linear Algebra with Applications*, 19(6):914–936, 2012.

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Thanks For Your
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